

Obtaining Parameters of a Battery Equivalent Circuit Model

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ABSTRACT

Good battery modeling is critical for energy management of electric vehicles and hybrid electric vehicles. Because of its simplicity and satisfactory performance, equivalent circuit models are widely used in this area. A frequently adopted equivalent circuit model is one that consists of an open-circuit voltage and a resistor in series with two sets of parallel resistor-capacitor combinations. This model performs well in describing battery transient behavior due to the dynamics of such physical phenomena as mass transport effects and double layer effects.

Generic methods for obtaining the parameters of this model involve analyzing the battery voltage behavior under step changes of load current. The fact that the model has two time constants places a challenge on parameter identification. Some most often used method makes use of the property that each of the two time constants plays a dominant role at different stages of the battery voltage response, and calculates the model parameters accordingly. For such method, the results are greatly influenced by the partition of faster and slower dynamics of the battery, and selection of the data points used for the calculation. Moreover, because majority of the testing data is not used towards calculating the parameters, the obtained model might not reflect the overall battery characteristics well and consequently might not give high-fidelity predictions. For other methods that use nonlinear curve fitting or genetic algorithm for parameter searching, the successful implementation greatly depends on the proper setting of initial values and searching space.

A novel method of parameter identification for the equivalent circuit model is presented in this paper. It makes use of a regression equation which is linear in variables that can be measured or calculated from the test. With this approach, all testing data during the relaxation period of a constant current pulse discharge or charge test is used towards obtaining the model parameters and the calculation is in the sense of least squared error. Application of the method to real battery testing data is presented. The example indicates that the method gives very good results with modeling error of less than 0.5%.

INTRODUCTION

With ever growing concerns on environmental issues and the depletion of fossil fuel supply, the automotive industry has put plenty of effort into the development of Electric Vehicles (EV) and Hybrid Electric Vehicles (HEV) [1]. For both forms of implementation the battery plays a critical role in affecting the overall vehicle performance [2]. A well designed energy management system, including battery management is fundamental for obtaining the highest possible emission reduction, significant fuel economy improvement, and better vehicle performance. The battery management functions monitor the battery voltage, current and temperature etc., estimate the status of the battery, and perform appropriate actions including cell balancing and thermal management etc., to guarantee that the battery is operating in the proper range for optimum vehicle performance.

Being able to accurately calculate the battery state of charge (SOC) over wide range of operating conditions is the prerequisite for almost all battery management functions [3,4]. To determine battery SOC, the relationship among SOC, battery open circuit voltage (OCV), and battery input to output dynamic characteristics, i.e., battery voltage v.s. current behavior, needs to be established. There are various approaches for setting up this mathematical representation, two major types of which are electrochemical modeling [5-7] and equivalent circuit modeling [8-16]. Electrochemical models are based on the battery physical construction and chemistry. While these models can be extremely accurate in describing the battery behavior, usually they are computationally time-consuming and not suitable for real-time control oriented applications. Different from the electrochemical approach, the equivalent circuit model

describes the effects of the chemical processes of major interest using electrical circuit elements, such as voltage source, resistors and capacitors etc. to approximate the battery dynamics.

Since battery dynamic effects can range as wide as from MHz to mHz, i.e., the time domain response can span from microseconds up to hours, days and even longer [17], one needs to pay attention to the targeted time range of the battery behavior when using the equivalent circuit model approach. For example, while the electromagnetic effects are very fast, such long-term effects as the cycling effect, reversible effect and aging effect have time constants from hours to years. The equivalent circuit model discussed in this paper is often used to describe the influence of such electrochemical phenomena as double layer effects and mass transport effects, which have time ranges from milliseconds to hours and have big influences on battery performance in EV and HEV applications.

Something deserves noting is although a physical explanation can be designated between the electric circuit components of the model and the actual battery dynamics effects, an equivalent circuit model is constructed mainly to match experimental data. Such a model, while simple and intuitive, provides sufficiently accurate information that serves the need of real-time battery management when correctly designed.

The widely used equivalent circuit model [11-16] studied in this paper is shown in figure 1. It consists of battery OCV V_{oc} , ohmic resistance of the connectors, electrodes and electrolyte R_0 , and two sets of parallel resistor-capacitor combination R_1, C_1 and R_2, C_2 representing the mass transport effects and the double layer effects respectively [13,15]. Commonly the time constants of the two dynamics differ by at least an order of magnitude.

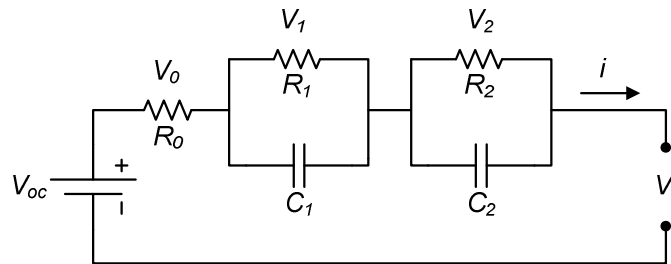


Figure 1 - Battery Equivalent Circuit Model.

To establish battery models of good quality, model parameters need be accurately calculated from test data. To identify the parameters, usually specific tests, such as constant current pulse discharge and charge tests at various SOC and current rates are performed. The ohmic resistance R_0 can be calculated easily based on the immediate battery voltage change before and after the current step [13-15]. However, the other parameters (R_1, C_1 and R_2, C_2) are more difficult to identify because two time constants are involved in the model. Some paper takes advantage of the fact that the two time constants differ greatly in magnitude, thus partitions the test data into the short time constant dominant segment and the long time constant dominant segment, and uses the battery voltage response in each segment to estimate the model parameters [13]. However, partition of the segments, and selection of the data points used for the calculation could be very arbitrary and thus the parameter values determined might vary greatly, and good performance of the model is not guaranteed. Moreover, since majority of the testing data except the selected data points is not used in the calculation, the overall battery characteristics might not be well reflect and consequently the model might not give high-fidelity predictions. Some other papers make use of the nonlinear curve fitting techniques [14] or genetic algorithm for searching of the parameters inside the defined space [16]. While these methods might lead to model parameters of good quality, they need users to have good experience in setting up the initial parameter values and the searching space.

This paper presents a novel parameter identification method for this battery equivalent circuit model. Different from the above mentioned approaches, this method makes use of a regression equation which is linear in variables that can be measured or calculated from the test, and takes into account all testing data during the relaxation period of constant current pulse tests towards obtaining the model parameters. Application to real testing data indicates that the method gives good results and the modeling error is less than 0.5%.

The paper is organized as follows. First, a detailed description of the parameter identification method is given. Then the test setup and procedure for obtaining the data needed for model parameter identification is described. Next, application of the designed method on the obtained testing data is presented, and the result is analyzed. The last section summarizes the paper.

PARAMETER IDENTIFICATION

Parameter identification of the battery equivalent circuit model includes determination of the battery OCV, the ohmic resistance, and the parallel resistor-capacitor parameters at various SOC. The tests performed are usually constant current pulse discharge or charge tests. Since the model parameters are functions of battery SOC, for parameter identification at each SOC, the discharge or charge pulse is set short so that the SOC, and all the model parameters can be considered constant during the testing.

While the method for determining the battery OCV and the ohmic resistance R_0 is the same as in several papers mentioned before [13-15], it is included here for completeness. As to identification of the remaining parameters (R_1, C_1 and R_2, C_2) that involve the two time constants, the approach presented here is based on establishing and making use of a regression equation which is linear in variables that can be measured or calculated from the test results.

Referring to Figure 1 where i is the current (positive for discharge and negative for charge), and V_t the battery voltage, it is evident:

$$V_t = V_{oc} - V_0 - V_1 - V_2 \quad (1)$$

$$V_0 = iR_0 \quad (2)$$

$$C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} = i \quad (3)$$

$$C_2 \frac{dV_2}{dt} + \frac{V_2}{R_2} = i \quad (4)$$

The two time constants of the system are:

$$\tau_1 = R_1 C_1 \quad (5)$$

$$\tau_2 = R_2 C_2 \quad (6)$$

Take a discharge test for example. As shown in Figure 2, assume the battery has been at rest for sufficiently long before a step change of magnitude I occurs for the current i at $t = T_0 = 0$. The corresponding battery voltage has an immediate drop at time T_0 due to the ohmic resistance. Then it enters the smoothly dropping portion which is governed by the time constants of the two resistor-capacitor combinations. When the current pulse ends at $t = T_1$, the battery voltage has an immediate jump, again due to the ohmic resistance, and then enters the relaxation stage where the voltage levels off exponentially to the OCV. Assume the test data is recorded until $t = T_2$.

A. Identification of V_{oc} and R_0

For determination of V_{oc} , since the battery has been at rest for sufficiently long and the test starts from zero state, we have:

$$V_{oc} = V_t(T_0^-) \quad (7)$$

As to R_0 , since the immediate battery voltage change when the current step occurs is due to the ohmic resistance, it can be calculated as:

$$R_0 = \frac{V_i(T_0^-) - V_i(T_0^+)}{I} \quad (8)$$

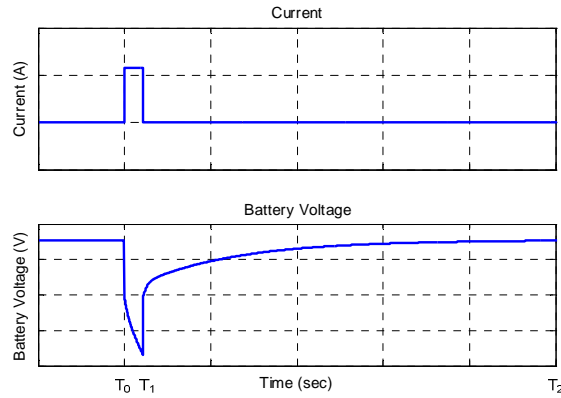


Figure 2 – Constant Current Pulse Discharge Test.

B. Identification of τ_1 and τ_2

To determine the remaining parameters, the time constants of the two resistor-capacitor combinations are calculated first. Since the pulse duration is quite short, the test data during the pulse might not contain sufficient information for correctly extracting these two time constants. On the contrary, the relaxation period may last minutes or even hours, thus the battery voltage behavior during the relaxation gives the best indication of the dominant time constants of the battery. Based on this observation, the test data from time T_1^+ to time T_2 is used for the identification of τ_1 and τ_2 . During this period the circuit is in zero input response. Denote the initial states as:

$$V_{10} = V_1(T_1) \quad (9)$$

$$V_{20} = V_2(T_1) \quad (10)$$

Then from (3) and (4) we have:

$$V_1(t - T_1) = V_{10} e^{-\frac{t - T_1}{\tau_1}} \quad (11)$$

$$V_2(t - T_1) = V_{20} e^{-\frac{t - T_1}{\tau_2}} \quad (12)$$

Since $i = 0$, (1) and (2) gives:

$$V_i(t - T_1) = V_{oc} - V_1(t - T_1) - V_2(t - T_1) \quad (13)$$

Define

$$U = V_1 + V_2 \quad (14)$$

This gives:

$$U(t-T_1) = V_{10}e^{-\frac{t-T_1}{\tau_1}} + V_{20}e^{-\frac{t-T_1}{\tau_2}} \quad (15)$$

While from (13) it is also obtained:

$$U(t-T_1) = V_{oc} - V_i(t-T_1) \quad (16)$$

Since both terms on the right side of equation (16) are available, $U(t-T_1)$ becomes a known variable.

To establish a regression equation that can be used for parameter identification, the following two variables are defined and calculated from the test data:

$$X(t-T_1) = \int_{T_1}^t U(\tau-T_1)d\tau \quad (17)$$

$$Y(t-T_1) = \int_{T_1}^t X(\tau-T_1)d\tau \quad (18)$$

By using (15) and (17), it is obtained:

$$X(t-T_1) = (V_{10}\tau_1 + V_{20}\tau_2) - (V_{10}\tau_1 e^{-\frac{t-T_1}{\tau_1}} + V_{20}\tau_2 e^{-\frac{t-T_1}{\tau_2}}) \quad (19)$$

Subsequently by using (18) and (19), we have:

$$\begin{aligned} Y(t-T_1) &= (V_{10}\tau_1 + V_{20}\tau_2)(t-T_1) - (V_{10}\tau_1^2 + V_{20}\tau_2^2) \\ &+ (V_{10}\tau_1^2 e^{-\frac{t-T_1}{\tau_1}} + V_{20}\tau_2^2 e^{-\frac{t-T_1}{\tau_2}}) \end{aligned} \quad (20)$$

By using any two of the three equations (15), (19) and (20), we can express $e^{-\frac{t-T_1}{\tau_1}}$ and $e^{-\frac{t-T_1}{\tau_2}}$ in other terms. When these expressions are substituted into the third equation, a regression equation is obtained as follows, which has no the exponential terms in it:

$$\begin{aligned} Y(t-T_1) &= -X(t-T_1)(\tau_1 + \tau_2) - U(t-T_1)\tau_1\tau_2 \\ &+ (V_{10}\tau_1 + V_{20}\tau_2)(t-T_1) + (V_{10} + V_{20})\tau_1\tau_2 \end{aligned} \quad (21)$$

Note in this equation U , X , and Y are all available from measurement and calculation. Expressing it in matrix format gives:

$$Y(t-T_1) = \begin{bmatrix} -X(t-T_1) & -U(t-T_1) & t-T_1 & 1 \end{bmatrix} \begin{bmatrix} \tau_1 + \tau_2 \\ \tau_1\tau_2 \\ V_{10}\tau_1 + V_{20}\tau_2 \\ (V_{10} + V_{20})\tau_1\tau_2 \end{bmatrix} \quad (22)$$

This is used for identification of the time constants τ_1 and τ_2 , as well as the initial states V_{10} and V_{20} . Take all the sampling points $t_k, k = 1, 2, \dots, n$ during the period from T_1^+ to T_2 and obtain:

$$U_k = U(t_k - T_1), k = 1, 2, \dots, n$$

$$X_k = X(t_k - T_1), k = 1, 2, \dots, n$$

$$Y_k = Y(t_k - T_1), k = 1, 2, \dots, n$$

Then from (22) we have

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \dots \\ Y_n \end{bmatrix} = \begin{bmatrix} -X_1 & -U_1 & t_1 - T_1 & 1 \\ -X_2 & -U_2 & t_2 - T_1 & 1 \\ \dots & \dots & \dots & \dots \\ -X_n & -U_n & t_n - T_1 & 1 \end{bmatrix} \begin{bmatrix} \tau_1 + \tau_2 \\ \tau_1 \tau_2 \\ V_{10} \tau_1 + V_{20} \tau_2 \\ (V_{10} + V_{20}) \tau_1 \tau_2 \end{bmatrix} \quad (23)$$

Denote

$$B = \begin{bmatrix} Y_1 \\ Y_2 \\ \dots \\ Y_n \end{bmatrix} \quad (24)$$

$$A = \begin{bmatrix} -X_1 & -U_1 & t_1 - T_1 & 1 \\ -X_2 & -U_2 & t_2 - T_1 & 1 \\ \dots & \dots & \dots & \dots \\ -X_n & -U_n & t_n - T_1 & 1 \end{bmatrix} \quad (25)$$

$$P = \begin{bmatrix} \tau_1 + \tau_2 \\ \tau_1 \tau_2 \\ V_{10} \tau_1 + V_{20} \tau_2 \\ (V_{10} + V_{20}) \tau_1 \tau_2 \end{bmatrix} \quad (26)$$

The equation (23) is

$$B = AP \quad (27)$$

and the least squared error solution is

$$P = (A^T A)^{-1} A^T B \quad (28)$$

After the values of P are obtained, τ_1, τ_2 and V_{10}, V_{20} can be calculated from (26).

C. Identification of R_1, C_1 and R_2, C_2

After the time constants τ_1, τ_2 and the initial states V_{10}, V_{20} are identified, R_1, C_1 and R_2, C_2 can be calculated as follows.

Consider the battery voltage behavior during the pulse. From (3) and (4) it is obtained that for $t \in [T_0, T_1]$

$$V_1(t) = IR_1(1 - e^{-\frac{t}{\tau_1}}) \quad (29)$$

$$V_2(t) = IR_2(1 - e^{-\frac{t}{\tau_2}}) \quad (30)$$

Thus at time T_1

$$V_1(T_1) = IR_1(1 - e^{-\frac{T_1}{\tau_1}}) \quad (31)$$

$$V_2(T_1) = IR_2(1 - e^{-\frac{T_1}{\tau_2}}) \quad (32)$$

which are exactly the initial states for the relaxation period. That is:

$$V_{10} = IR_1(1 - e^{-\frac{T_1}{\tau_1}}) \quad (33)$$

$$V_{20} = IR_2(1 - e^{-\frac{T_1}{\tau_2}}) \quad (34)$$

Since V_{10} and V_{20} are already obtained, R_1 and R_2 can be calculated from (33) and (34), and subsequently C_1 and C_2 can be calculated from (5) and (6).

Note τ_1 and τ_2 are obtained based on all the testing data during the relaxation period, while the subsequent identification of R_1, R_2 is mainly based on matching the initial states for the zero input response, thus the performance of the identified parameters in modeling the behavior during the pulse might not be as good as in modeling the behavior of the relaxation period.

Also deserving of note is that the model parameters for charging and discharging will differ at each SOC. However, the designed parameter identification method works for both cases.

APPLICATION EXAMPLE

To study the performance of the designed parameter identification method, it is applied to the testing results of a 1.2 V, 1.22 Ah NiMH battery cell. The test setup is as shown in Figure 3. The battery cell under test is connected to a programmable cycler. A data acquisition and control system communicates with the cycler for command setting as well as battery current and voltage sampling. The system also samples the temperature of the battery. Sampling rate is set to 100 Hz for the tests.

Tests are performed for both discharge and charge, and various SOC. Before each test, the battery has been at rest for at least 1 hour. At the start of each test the battery voltage is recorded, which is considered as the OCV at this SOC. Then a constant discharge or charge current of about 1C is applied. The current pulse lasts about 20 seconds and then the battery returns to idle period again. The tests are performed in the SOC range between 90% and 10%, in 10% intervals.

Take as an example the results of a test at 50% SOC using a discharge current of 1.15 A, and pulse duration of 21.4 seconds. The OCV is 1.2771 V. When the discharge pulse starts, a battery voltage drop of 0.0410 V is observed. This gives the value of R_0 of 0.0356 ohm. The test data is sampled for over 2500 seconds, including the pre-pulse, during pulse, and relaxation stages. From all the data during the relaxation period, it is identified that the two time constants are $\tau_1 = 1109.7$ sec, $\tau_2 = 45.1$ sec, and the initial states are obtained as $V_{10} = 0.0066$ V, $V_{20} = 0.0075$ V. R_1, R_2 and C_1, C_2 are subsequently identified using the designed method and the results are $R_1 = 0.2988$ ohm, $R_2 = 0.0173$ ohm, $C_1 = 3713.6$ F, $C_2 = 2607.5$ F.

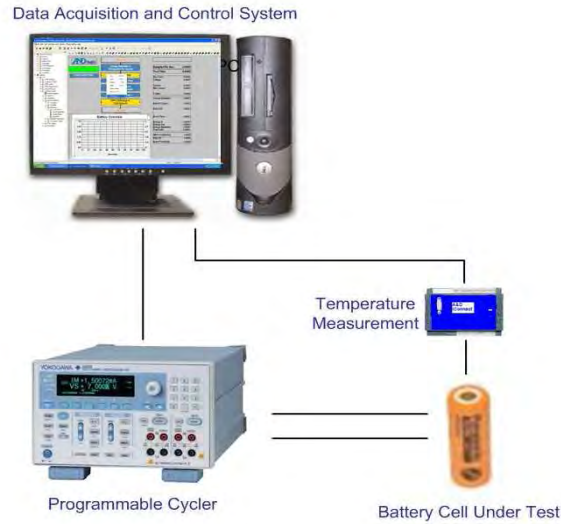


Figure 3 – Battery Test Setup.

Comparison of the model output and the measured results is shown in Figure 4 for the whole test period. Figure 5 shows the results for 200 seconds to give more details of the performance. It is evident the model matches the testing data very well and the maximum error between the measured and calculated voltage is only 0.0023 V, or 0.18% of the OCV at this SOC.

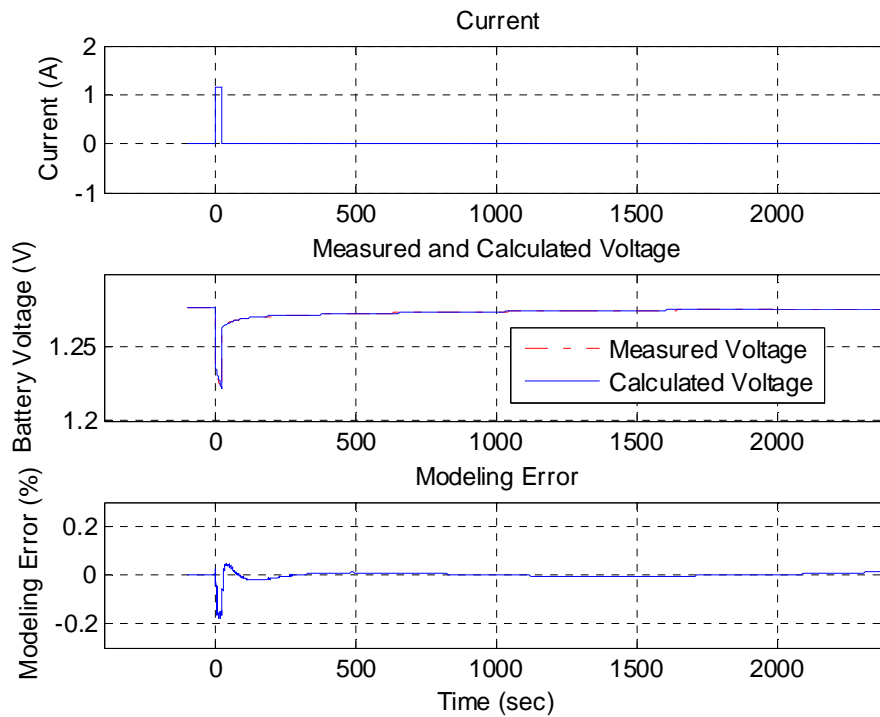


Figure 4 – Measured and Calculated Battery Voltage.

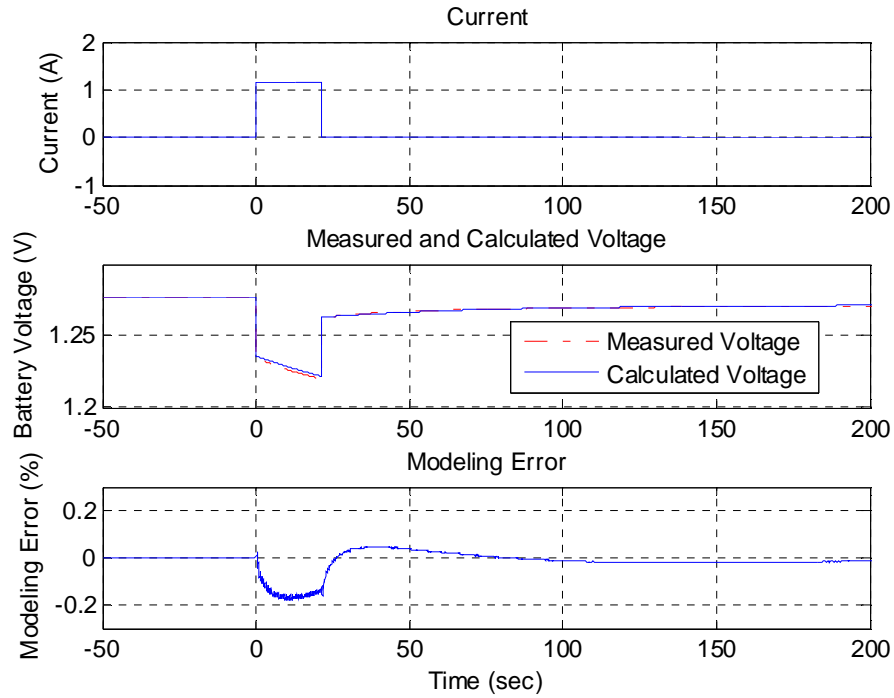


Figure 5 – Details of Measured and Calculated Battery Voltage (for 200 Seconds).

The model parameters are functions of SOC. Parameter identification results for various SOC levels are summarized as in Table 1. In addition to all the parameter values, the maximum modeling error, both in volt and in percentage of the corresponding OCV, is included in the table as well. It can be seen that for the SOC range from 20% to 90%, all the modeling errors are less than 0.5%. While for the 10% SOC, the error is as big as 2.8%. This may indicate that at very low level of SOC, the structure of the equivalent circuit model might not be able to reflect the battery behavior well. Relationship of the model parameters with respect to SOC is shown in Figure 6. Note that because some parameters deviate greatly in magnitude, for display purpose a factor of 10 is used for those with smaller values.

Table 1 – Identified Battery Model Parameters.

SOC (%)	V_{oc} (V)	R_0 (ohm)	R_1 (ohm)	R_2 (ohm)	C_1 (F)	C_2 (F)	τ_1 (sec)	τ_2 (sec)	Max error (V)	Max error (%)
10	1.2066	0.0514	1.0820	0.1169	1132.9	409.9	1225.8	47.9	0.0338	2.80
20	1.2360	0.0398	0.5886	0.0384	2015.9	1196.8	1186.6	45.9	0.0056	0.46
30	1.2511	0.0373	0.4083	0.0232	2577.8	1247.9	1052.5	28.9	0.0024	0.20
40	1.2644	0.0361	0.3384	0.0191	3220.2	2060.1	1089.7	39.3	0.0023	0.19
50	1.2771	0.0356	0.2988	0.0173	3713.6	2607.5	1109.7	45.1	0.0023	0.18
60	1.2884	0.0366	0.2576	0.0145	3802.9	1930.0	979.8	27.9	0.0014	0.11
70	1.2989	0.0380	0.2963	0.0164	4084.0	1989.2	1210.1	32.6	0.0014	0.11
80	1.3136	0.0384	0.4270	0.0239	4496.4	3142.4	1920.1	75.2	0.0035	0.27
90	1.3461	0.0388	1.6769	0.0320	4206.2	2887.5	7053.3	92.3	0.0041	0.31

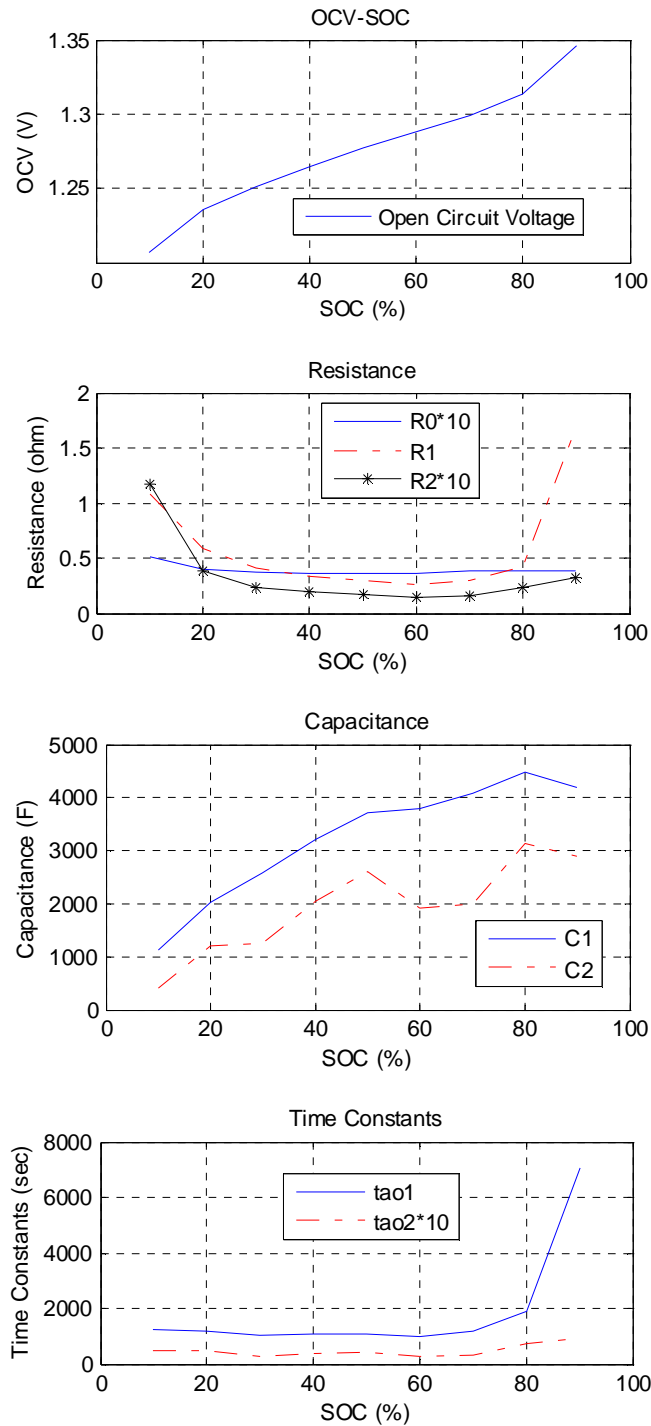


Figure 6 – Model Parameters as Functions of SOC .

SUMMARY/CONCLUSIONS

It is important to accurately identify the parameters of a battery model in order to fulfill good battery management functions. A novel method which makes use of a regression equation that is linear in variables that can be measured or calculated from battery test is designed for the parameter identification of a widely used battery equivalent circuit model. With this approach, all testing data during the relaxation period of a constant current pulse discharge or charge test is used towards obtaining the model parameters and the

calculation is in the sense of least squared error. Application of the method to real battery testing data shows that the method gives very good modeling performance and the error is less than 0.5%.

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